Problem 9.38  A three-element linear array of isotropic sources aligned along the $z$-axis has an interelement spacing of $\lambda/4$ (Fig. P9.38). The amplitude excitation of the center element is twice that of the bottom and top elements, and the phases are $-\pi/2$ for the bottom element and $\pi/2$ for the top element, relative to that of the center element. Determine the array factor and plot it in the elevation plane.

**Figure P9.38:** Three-element array of Problem 9.38.

**Solution:** From Eq. (9.110),

$$F_a(\theta) = \left| \sum_{i=0}^{2} a_i e^{j\psi_i} e^{ikd\cos\theta} \right|^2$$

$$= |a_0 e^{j\psi_0} + a_1 e^{j\psi_1} e^{ikd\cos\theta} + a_2 e^{j\psi_2} e^{ikd\cos\theta}|^2$$

$$= |e^{j(\psi_1-\pi/2)} + 2e^{j\psi_1} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta} + e^{j(\psi_1+\pi/2)} e^{j(2\pi/\lambda)(\lambda/4)\cos\theta}|^2$$

$$= |e^{j\psi_1} e^{j(\pi/2)\cos\theta}|^2 |e^{-j\pi/2} e^{-j(\pi/2)\cos\theta} + 2 + e^{j\pi/2} e^{j(\pi/2)\cos\theta}|^2$$

$$= 4(1 + \cos(\frac{1}{4}(1 + \cos\theta)))^2,$$

$$F_{an}(\theta) = \frac{1}{4}(1 + \cos(\frac{1}{4}(1 + \cos\theta)))^2.$$  

This normalized array factor is shown in Fig. 9.38(b).
Figure P9.38: (b) Normalized array pattern of the 3-element array of Problem 9.38.