Problem 8.6  A 50-MHz plane wave with electric field amplitude of 50 V/m is normally incident in air onto a semi-infinite, perfect dielectric medium with $\varepsilon_r = 36$. Determine the following:

(a) $\Gamma$
(b) The average power densities of the incident and reflected waves.
(c) The distance in the air medium from the boundary to the nearest minimum of the electric field intensity, $|E|$.

Solution:

(a) 
\begin{align*}
\eta_1 &= \eta_0 = 120\pi \text{ (}\Omega), \\
\eta_2 &= \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{120\pi}{\varepsilon_2} = \frac{120\pi}{6} = 20\pi \text{ (}\Omega), \\
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi - 120\pi}{20\pi + 120\pi} = -0.71.
\end{align*}

Hence, $|\Gamma| = 0.71$ and $\theta_\eta = 180^\circ$.

(b) 
\begin{align*}
S^i_{av} &= \frac{|E^i_0|^2}{2\eta_1} = \frac{(50)^2}{2 \times 120\pi} = 3.32 \text{ (W/m}^2), \\
S^r_{av} &= |\Gamma|^2 S^i_{av} = (0.71)^2 \times 3.32 = 1.67 \text{ (W/m}^2).
\end{align*}

(c) In medium 1 (air),
\begin{align*}
\lambda_1 &= \frac{c}{f} = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}.
\end{align*}

From Eqs. (8.16) and (8.17),
\begin{align*}
l_{\text{max}} &= \frac{\theta_\lambda \lambda_1}{4\pi} = \frac{\pi \times 6}{4\pi} = 1.5 \text{ m}, \\
l_{\text{min}} &= l_{\text{max}} - \frac{\lambda_1}{4} = 1.5 - 1.5 = 0 \text{ m (at the boundary).}
\end{align*}