Problem 8.47  A hollow cavity made of aluminum has dimensions \( a = 4 \) cm and \( d = 3 \) cm. Calculate \( Q \) of the TE\(_{101}\) mode for

(a) \( b = 2 \) cm, and

(b) \( b = 3 \) cm.

Solution:

For the TE\(_{101}\) mode, \( f_{101} \) is independent of \( b \),

\[
f_{101} = \frac{c}{2} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{1}{d} \right)^2}
\]

\[
= \frac{3 \times 10^8}{1} \sqrt{\left( \frac{1}{4 \times 10^{-2}} \right)^2 + \left( \frac{1}{3 \times 10^{-2}} \right)^2}
\]

\[
= 6.25 \text{ GHz.}
\]

For aluminum with \( \sigma_c = 3.5 \times 10^7 \) S/m (Appendix B),

\[
\delta_k = \frac{1}{\sqrt{\pi f_{101} \mu_0 \sigma_c}} = 1.08 \times 10^{-6} \text{ m.}
\]

(a) For \( a = 4 \) cm, \( b = 2 \) cm and \( d = 3 \) cm,

\[
Q = \frac{1}{\delta_k} \frac{abd(a^2 + d^2)}{[a^3(d + 2b) + d^3(a + 2b)]}
\]

\[
= 8367.
\]

(b) For \( a = 4 \) cm, \( b = 3 \) cm, and \( d = 3 \) cm,

\[
Q = 9850.
\]