Problem 7.7  A 60-MHz plane wave traveling in the $-x$-direction in dry soil with relative permittivity $\varepsilon_r = 4$ has an electric field polarized along the $z$-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave’s electric and magnetic fields.

Solution: For $f = 60$ MHz $= 6 \times 10^7$ Hz, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c} \sqrt{\varepsilon_r} = \frac{2\pi \times 6 \times 10^7}{3 \times 10^8} \sqrt{4} = 0.8\pi \quad \text{(rad/m)}.$$

Given that $\mathbf{E}$ points along $\hat{z}$ and wave travel is along $-\hat{x}$, we can write

$$\mathbf{E}(x, t) = \hat{z} E_0 \cos(2\pi \times 60 \times 10^6 t + 0.8\pi x + \phi_0) \quad \text{(V/m)}$$

where $E_0$ and $\phi_0$ are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \quad \text{(Ω)}.$$

With $\mathbf{E}$ along $\hat{z}$ and $\mathbf{H}$ along $-\hat{x}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta} \hat{k} \times \mathbf{E}$$

or

$$\mathbf{H}(x, t) = \hat{y} \frac{E_0}{\eta} \cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad \text{(A/m)}.$$

Hence,

$$\frac{E_0}{\eta} = 10 \quad \text{(mA/m)}$$

$$E_0 = 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad \text{(V/m)}.$$

Also,

$$H(-0.75 \text{ m}, 0) = 7 \times 10^{-3} = 10 \cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 153.6^\circ$.

Hence,

$$\mathbf{E}(x, t) = \hat{z} 0.6\pi \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(V/m)}.$$  

$$\mathbf{H}(x, t) = \hat{y} 10 \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6^\circ) \quad \text{(mA/m)}.$$