Problem 4.53  Dielectric breakdown occurs in a material whenever the magnitude of the field \( E \) exceeds the dielectric strength anywhere in that material. In the coaxial capacitor of Example 4-12,

(a) At what value of \( r \) is \( |E| \) maximum?

(b) What is the breakdown voltage if \( a = 1 \) cm, \( b = 2 \) cm, and the dielectric material is mica with \( \varepsilon_r = 6 \)?

Solution:

(a) From Eq. (4.114), \( E = -\hat{\mathbf{r}} \rho_l / 2\pi \varepsilon r \) for \( a < r < b \). Thus, it is evident that \( |E| \) is maximum at \( r = a \).

(b) The dielectric breaks down when \( |E| > 200 \) (MV/m) (see Table 4-2), or

\[
|E| = \frac{\rho_l}{2\pi \varepsilon r} = \frac{\rho_l}{2\pi (6\varepsilon_0)(10^{-12})} = 200 \text{ (MV/m)},
\]

which gives \( \rho_l = (200 \text{ MV/m})(2\pi)(8.854 \times 10^{-12})(0.01) = 667.6 \text{ (\mu C/m)} \).

From Eq. (4.115), we can find the voltage corresponding to that charge density,

\[
V = \frac{\rho_l}{2\pi \varepsilon} \ln \left( \frac{b}{a} \right) = \frac{(667.6 \text{ \mu C/m})}{12\pi (8.854 \times 10^{-12} \text{ F/m})} \ln(2) = 1.39 \text{ (MV)}.
\]

Thus, \( V = 1.39 \) (MV) is the breakdown voltage for this capacitor.