Problem 4.21 A horizontal strip lying in the $x$–$y$ plane is of width $d$ in the $y$-direction and infinitely long in the $x$-direction. If the strip is in air and has a uniform charge distribution $\rho_s$, use Coulomb’s law to obtain an explicit expression for the electric field at a point $P$ located at a distance $h$ above the centerline of the strip. Extend your result to the special case where $d$ is infinite and compare it with Eq. (4.25).

Solution:

![Diagram of a horizontal strip of charge](image)

Figure P4.21: Horizontal strip of charge.

The strip of charge density $\rho_s$ (C/m$^2$) can be treated as a set of adjacent line charges each of charge $\rho_l = \rho_s \, dy$ and width $dy$. At point $P$, the fields of line charge at distance $y$ and line charge at distance $-y$ give contributions that cancel each other along $\hat{y}$ and add along $\hat{z}$. For each such pair,

$$dE = 2 \frac{\rho_s \, dy \, \cos \theta}{2\pi \varepsilon_0 R}.$$

With $R = h / \cos \theta$, we integrate from $y = 0$ to $d/2$, which corresponds to $\theta = 0$ to
\[ \theta_0 = \sin^{-1}\left[\frac{d/2}{h^2 + (d/2)^2}^{1/2}\right]. \] Thus,

\[ E = \int_0^{d/2} dE = \frac{2}{\pi\varepsilon_0} \int_0^{d/2} \frac{\cos \theta}{R} \, dy = \frac{2}{\pi\varepsilon_0} \int_0^{\theta_0} \cos^2 \theta \, \frac{h}{\cos^2 \theta} \, d\theta = \frac{2}{\pi\varepsilon_0} \theta_0. \]

For an infinitely wide sheet, \( \theta_0 = \pi/2 \) and \( E = \frac{2}{2\varepsilon_0} \rho_s \), which is identical with Eq. (4.25).