**Problem 5.22** A long cylindrical conductor whose axis is coincident with the \(z\)-axis has a radius \(a\) and carries a current characterized by a current density \(\mathbf{J} = \hat{\mathbf{z}} J_0 / r\), where \(J_0\) is a constant and \(r\) is the radial distance from the cylinder’s axis. Obtain an expression for the magnetic field \(\mathbf{H}\) for

(a) \(0 \leq r \leq a\)

(b) \(r > a\)

**Solution:** This problem is very similar to Example 5-5.

(a) For \(0 \leq r_1 \leq a\), the total current flowing within the contour \(C_1\) is

\[
I_1 = \int \int \mathbf{J} \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{r=0}^{r_1} \left( \frac{\hat{\mathbf{z}} J_0}{r} \right) \cdot \hat{\mathbf{z}} r dr d\phi = 2\pi \int_{r=0}^{r_1} J_0 dr = 2\pi r_1 J_0.
\]

Therefore, since \(I_1 = 2\pi r_1 H_1\), \(H_1 = J_0\) within the wire and \(H_1 = \hat{\mathbf{z}} J_0\).

(b) For \(r \geq a\), the total current flowing within the contour is the total current flowing within the wire:

\[
I = \int \int \mathbf{J} \cdot d\mathbf{s} = \int_{\phi=0}^{2\pi} \int_{r=0}^{a} \left( \frac{\hat{\mathbf{z}} J_0}{r} \right) \cdot \hat{\mathbf{z}} r dr d\phi = 2\pi \int_{r=0}^{a} J_0 dr = 2\pi a J_0.
\]

Therefore, since \(I = 2\pi r H_2\), \(H_2 = J_0 a / r\) within the wire and \(H_2 = \hat{\mathbf{z}} J_0 (a / r)\).