**Problem 5.14**  Two parallel, circular loops carrying a current of 40 A each are arranged as shown in Fig. P5.14. The first loop is situated in the \( x-y \) plane with its center at the origin, and the second loop’s center is at \( z = 2 \) m. If the two loops have the same radius \( a = 3 \) m, determine the magnetic field at:

(a) \( z = 0 \)
(b) \( z = 1 \) m
(c) \( z = 2 \) m

**Solution:** The magnetic field due to a circular loop is given by (5.34) for a loop in the \( x-y \) plane carrying a current \( I \) in the \(+\hat{\phi}\) direction. Considering that the bottom loop in Fig. is in the \( x-y \) plane, but the current direction is along \(-\hat{\phi}\),

\[
H_1 = -\hat{z} \frac{Ia^2}{2(a^2 + z^2)^{3/2}},
\]

where \( z \) is the observation point along the \( z \)-axis. For the second loop, which is at a height of 2 m, we can use the same expression but \( z \) should be replaced with \((z - 2)\). Hence,

\[
H_2 = -\hat{z} \frac{Ia^2}{2[a^2 + (z - 2)^2]^{3/2}}.
\]

The total field is

\[
H = H_1 + H_2 = -\hat{z} \frac{Ia^2}{2} \left[ \frac{1}{(a^2 + z^2)^{3/2}} + \frac{1}{[a^2 + (z - 2)^2]^{3/2}} \right] \text{ A/m.}
\]
(a) At $z = 0$, and with $a = 3$ m and $I = 40$ A,
\[
H = -\hat{z} \frac{40 \times 9}{2} \left[ \frac{1}{3^3} + \frac{1}{(9+4)^{3/2}} \right] = -\hat{z} 10.5 \text{ A/m}.
\]

(b) At $z = 1$ m (midway between the loops):
\[
H = -\hat{z} \frac{40 \times 9}{2} \left[ \frac{1}{(9+1)^{3/2}} + \frac{1}{(9+1)^{3/2}} \right] = -\hat{z} 11.38 \text{ A/m}.
\]

(c) At $z = 2$ m, $H$ should be the same as at $z = 0$. Thus,
\[
H = -\hat{z} 10.5 \text{ A/m}.
\]