**Problem 4.56** Figure P4.56(a) depicts a capacitor consisting of two parallel, conducting plates separated by a distance $d$. The space between the plates contains two adjacent dielectrics, one with permittivity $\varepsilon_1$ and surface area $A_1$ and another with $\varepsilon_2$ and $A_2$. The objective of this problem is to show that the capacitance $C$ of the configuration shown in Fig. P4.56(a) is equivalent to two capacitances in parallel, as illustrated in Fig. P4.56(b), with

$$C = C_1 + C_2 \quad (19)$$

where

$$C_1 = \frac{\varepsilon_1 A_1}{d} \quad (20)$$

$$C_2 = \frac{\varepsilon_2 A_2}{d} \quad (21)$$

To this end, proceed as follows:

(a) Find the electric fields $E_1$ and $E_2$ in the two dielectric layers.

(b) Calculate the energy stored in each section and use the result to calculate $C_1$ and $C_2$.

(c) Use the total energy stored in the capacitor to obtain an expression for $C$. Show that (19) is indeed a valid result.

[Diagram of capacitor with parallel dielectric section and equivalent circuit]
Solution:

\[ E_1 = E_2 = \frac{V}{d}. \]

\[ W_{e1} = \frac{1}{2} \varepsilon_1 E_1^2 \cdot V = \frac{1}{2} \varepsilon_1 \frac{V^2}{d^2} \cdot A_1 d = \frac{1}{2} \varepsilon_1 V^2 \frac{A_1}{d}. \]

But, from Eq. (4.121),

\[ W_{e1} = \frac{1}{2} C_1 V^2. \]

Hence \( C_1 = \varepsilon_1 \frac{A_1}{d} \). Similarly, \( C_2 = \varepsilon_2 \frac{A_2}{d} \).

(c) Total energy is

\[ W_e = W_{e1} + W_{e2} = \frac{1}{2} \frac{V^2}{d} \left( \varepsilon_1 A_1 + \varepsilon_2 A_2 \right) = \frac{1}{2} CV^2. \]

Hence,

\[ C = \frac{\varepsilon_1 A_1}{d} + \frac{\varepsilon_2 A_2}{d} = C_1 + C_2. \]