**Problem 4.26**  In a certain region of space, the charge density is given in cylindrical coordinates by the function:

\[ \rho_v = 5re^{-r} \quad (\text{C/m}^3) \]

Apply Gauss’s law to find \( \mathbf{D} \).

**Solution:**

![Gaussian surface](image)

**Figure P4.26:** Gaussian surface.

**Method 1: Integral Form of Gauss’s Law**

Since \( \rho_v \) varies as a function of \( r \) only, so will \( \mathbf{D} \). Hence, we construct a cylinder of radius \( r \) and length \( L \), coincident with the \( z \)-axis. Symmetry suggests that \( \mathbf{D} \) has the functional form \( \mathbf{D} = \hat{r} \mathbf{D} \). Hence,

\[
\oint_S \mathbf{D} \cdot d\mathbf{s} = Q,
\]

\[
\int \hat{r} \mathbf{D} \cdot d\mathbf{s} = D(2\pi rL),
\]

\[
Q = 2\pi L \int_0^r 50re^{-r} \cdot r \, dr
\]

\[
= 100\pi L[-r^2e^{-r} + 2(1 - e^{-r}(1 + r))],
\]

\[
\mathbf{D} = \hat{r} \mathbf{D} = \hat{r} 50 \left[ \frac{2}{r}(1 - e^{-r}(1 + r)) - re^{-r} \right].
\]
Method 2: Differential Method

\[ \nabla \cdot \mathbf{D} = \rho_v, \quad \mathbf{D} = \hat{r} D_r, \]

with \( D_r \) being a function of \( r \).

\[
\frac{1}{r} \frac{\partial}{\partial r} (r D_r) = 50 r e^{-r}, \\
\frac{\partial}{\partial r} (r D_r) = 50 r^2 e^{-r}, \\
\int_0^r \frac{\partial}{\partial r} (r D_r) \, dr = \int_0^r 50 r^2 e^{-r} \, dr, \\
r D_r = 50 \left[ 2(1 - e^{-r}(1 + r)) - r^2 e^{-r} \right], \\
\mathbf{D} = \hat{r} r D_r = \hat{r} \left[ \frac{2}{r} (1 - e^{-r}(1 + r)) - re^{-r} \right].
\]