Problem 2.16 A transmission line operating at 125 MHz has $Z_0 = 40 \, \Omega$, $\alpha = 0.02 \, (\text{Np/m})$, and $\beta = 0.75 \, \text{rad/m}$. Find the line parameters $R'$, $L'$, $G'$, and $C'$.

Solution: Given an arbitrary transmission line, $f = 125 \, \text{MHz}$, $Z_0 = 40 \, \Omega$, $\alpha = 0.02 \, \text{Np/m}$, and $\beta = 0.75 \, \text{rad/m}$. Since $Z_0$ is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega \sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \, \text{nH/m}.$$ 

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \, \text{nH/m}}{40^2} = 23.9 \, \text{pF/m}.$$ 

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'} \sqrt{\frac{R'}{G'}} = \sqrt{R'G'} \sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \, \text{Np/m} \times 40 \, \Omega = 0.6 \, \Omega/m$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \, \text{Np/m})^2}{0.8 \, \Omega/m} = 0.5 \, \text{mS/m}.$$